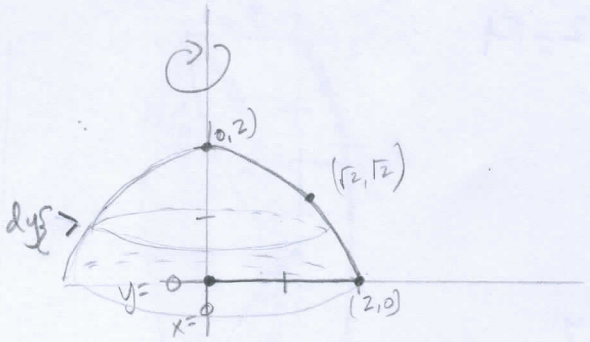
① $y=\sqrt{4-x^2}$, $x=0$, $y=0$ Revolved about y-axis $(x=0)$

$y^2=4-x^2$

$y^2-4=-x^2$

$\pm\sqrt{4-y^2}=x^2$

$x=\pm\sqrt{4-y^2}$



Using disc method

$$V=\pi\int_0^2(\sqrt{4-y^2})^2\,dy=\pi\int_0^2(4-y^2)\,dy=\pi\left(4y-\frac{y^3}{3}\right)\Big|_0^2$$

$$=\pi\left(4\cdot2-\frac{2^3}{3}\right)=\pi\left(\frac{24-8}{3}\right)=\boxed{\frac{16\pi}{3}\ u^3}$$

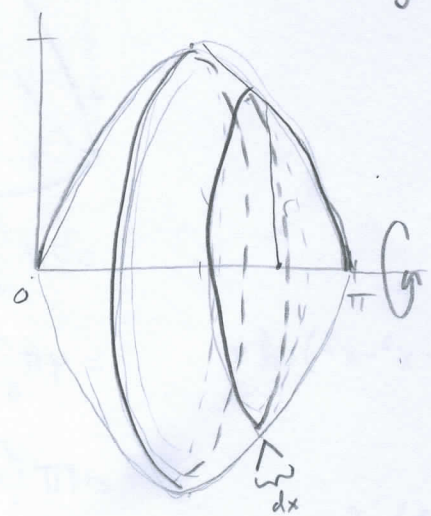
② $y=\sqrt{\sin x}$, $x=0$, $x=\pi$ Revolved about x-axis $(y=0)$

Disc Method

$$V=\pi\int_0^\pi(\sqrt{\sin x})^2\,dx=\pi\int_0^\pi(\sin x)\,dx$$

$$-\pi\left(\cos x\Big|_0^\pi\right)=-\pi(\cos\pi-\cos 0)$$

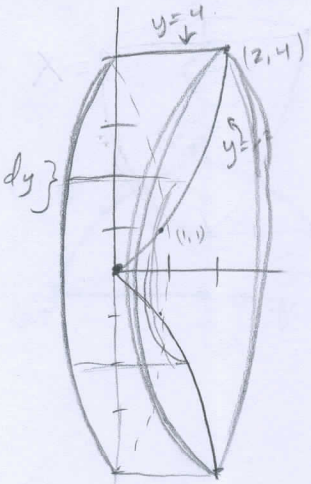
$$=-\pi(-1-1)=\boxed{2\pi\ u^3}$$



③ $y = x^2$ $y = 4$ $x = 0$ (y axis) Revolved about the x-axis ($y = 0$)

$\Downarrow$

$x = \sqrt{y}$ $\quad \sqrt{x^2} = \sqrt{4}$



Using Shell

$$V = 2\pi \int_0^4 (y)(y^{\frac{1}{2}})\, dy$$

$$= 2\pi \int_0^4 y^{\frac{3}{2}}\, dy$$

$$= 2\pi \left(\frac{2y^{\frac{5}{2}}}{5} \Big|_0^4 \right) = 2\pi \left(\frac{64}{5} \right) = \boxed{\frac{128\pi}{5}\ u^3}$$

---

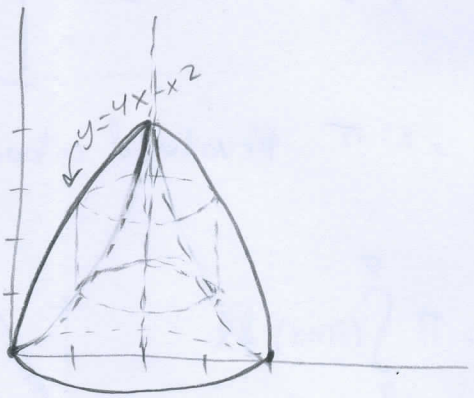
④ $y = x^2$, $y = 4x - x^2$ revolved about $x = 2$

$x^2 = 4x - x^2$ $\quad x = 0$, $x = 1 \cdot 2$

$\frac{2x^2}{4} = \frac{4x}{4}$ $\quad x = 2$

$\frac{1}{2}x^2 - x = 0$

$x(\frac{1}{2}x - 1) = 0$



Using Shell

$$V = 2\pi \int_0^2 (2-x)(4x - x^2 - x^2)\, dx$$

$$= 2\pi \int_0^2 (2-x)(4x - 2x^2)\, dx$$

$$= 2\pi \int_0^2 (8x - 4x^2 - 4x^2 + 2x^3)\, dx$$

$$= 2\pi \int_0^2 (2x^3 - 8x^2 + 8x)\, dx$$

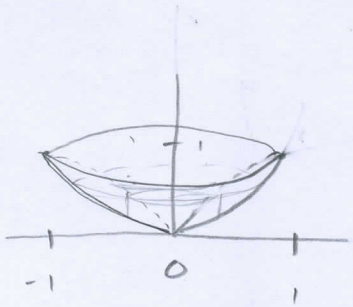
$$= 4\pi \int_0^2 (x^3 - 4x^2 + 4x)\, dx$$

$$= 4\pi \left(\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2 \Big|_0^2 \right)$$

$$= 4\pi \left(\frac{16}{4} - \frac{32}{3} + 8 \right) = 4\pi \left(\frac{48 - 128 + 96}{12\ 3} \right) = \boxed{\frac{16\pi}{3}\ u^3}$$

⑤ $y=\sqrt{x}$, $y=x^2$ revolved about y axis ($x=0$)

$y^2=x$ $\quad\quad$ $\sqrt{y}=x$



using shell,

$$V=2\pi\int_0^1 \left(x\left(x^{\frac{1}{2}}-x^2\right)\right)dx$$

$$=2\pi\int_0^1 \left(x^{\frac{3}{2}}-x^3\right)dx$$

$$=2\pi\left(\frac{2x^{\frac{5}{2}}}{5}-\frac{x^4}{4}\right)\Bigg|_0^1 = 2\pi\left(\frac{2}{5}-\frac{1}{4}\right)=8\pi\left(\frac{8-5}{10\,20}\right)=\boxed{\frac{3\pi}{10}\ u^3}$$

using washer

$$V=\pi\int_0^1 \left[(x^2)^2-(\sqrt{x})^2\right]dy$$

→ NOT OK AFTER THIS CLASS. HAHA.

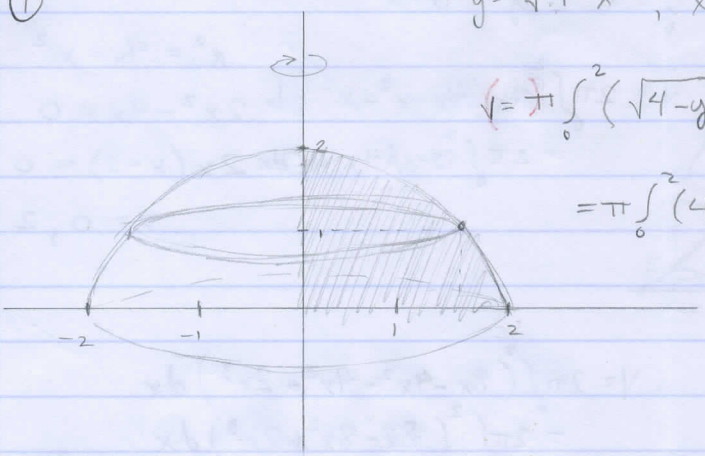
$$=\pi\int_0^1 \left[(\sqrt{y})^2-(y^2)^2\right]dy$$

$$=\pi\int_0^1 (y-y^4)\,dy$$

$$=\pi\left(\frac{y^2}{2}-\frac{y^5}{5}\right)\Bigg|_0^1 = \pi\left(\frac{1}{2}-\frac{1}{5}\right)=\pi\left(\frac{5-2}{10}\right)=\boxed{\frac{3\pi}{10}\ u^3}$$

Quiz #2 Math 152

① $y=\sqrt{4-x^2}$, $x=0$, $y=0$



$V=\pi\int_0^2\left(\sqrt{4-y^2}^2\right)dy$

$=\pi\int_0^2(4-y^2)\,dy$

$=\pi\left(4y-\frac{1}{3}y^3\right)\Big|_0^2=\pi\left(8-\frac{8}{3}\right)=\frac{16}{3}\pi\ u^3$

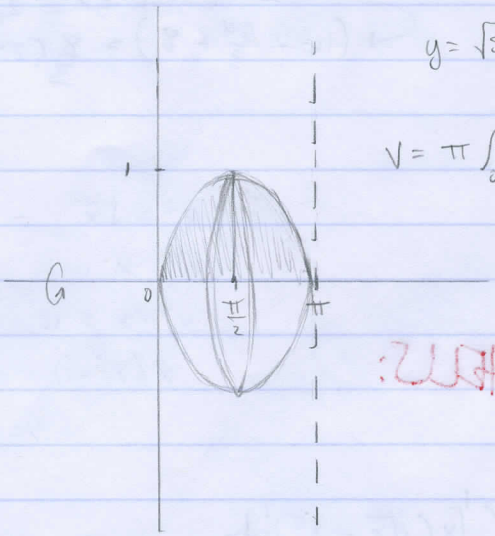
$\sqrt{4-x^2}=0$

$4-x^2=0$

$4=x^2$

$x=\pm 2$

② $y=\sqrt{\sin x}$, $x=0$, $x=\pi$



$V=\pi\int_0^{\pi}\left(\sqrt{\sin x}^2\right)dx$

$=\pi\int_0^{\pi}(\sin x)\,dx$

$=-\pi(\cos x\Big|_0^{\pi}=-1-(1)=-2$

NEGATIVE

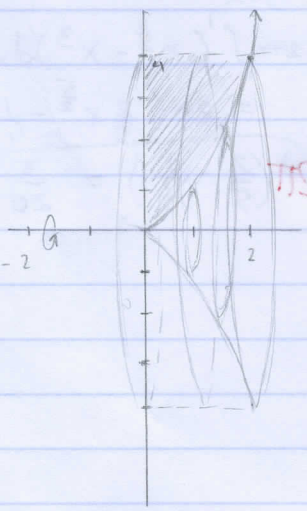
$=2\pi\ u^3$

$\sqrt{\sin x}=0$

$\sin x=0$

$x=0,\pi\ldots$

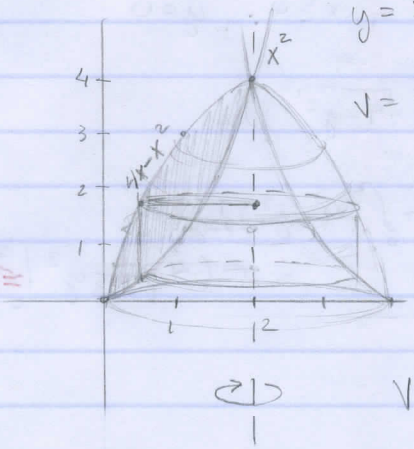
③ $y=x^2$ $x=\pm\sqrt{y}$



$V=2\pi\int_0^4 y(\sqrt{y})\,dy=2\pi\int_0^4\left(y^{\frac{3}{2}}\right)dy$

$=2\pi\left(\frac{2}{5}y^{\frac{5}{2}}\Big|_0^4=\frac{128\pi}{5}\ u^3\right.$

④



$y = x^2 \quad y = 4x - x^2$

$x^2 = 4x - x^2$

$V = 2\pi \int_0^2 [(2-x)(4x - x^2 - x^2)]\,dx \quad 2x^2 - 4x = 0$

$= 2\pi \int_0^2 (2-x)(4x - 2x^2)\,dx \quad 2x(x-2) = 0$

$x = 0, 2$

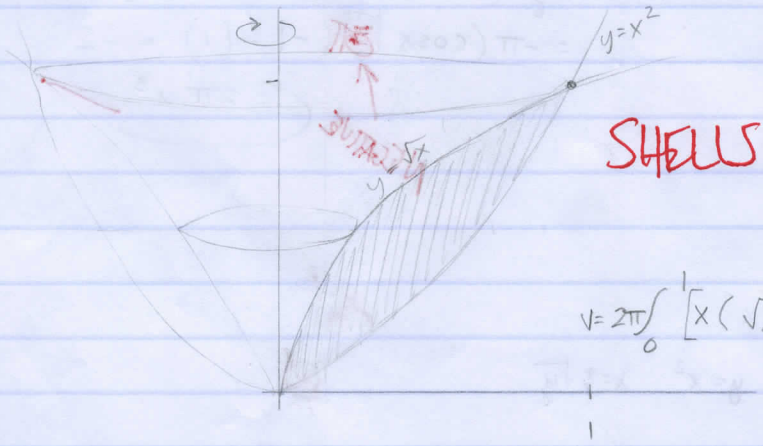
$V = 2\pi \int_0^2 (8x - 4x^2 - 4x^2 + 2x^3)\,dx$

$= 2\pi \int_0^2 (8x - 8x^2 + 2x^3)\,dx$

$= 2\pi \left(4x^2 - \frac{8}{3}x^3 + \frac{1}{2}x^4\right)\Big|_0^2$

$\rightarrow \left(16 - \frac{64}{3} + 8\right) = \frac{8}{3}(2\pi) = \frac{16\pi}{3}\ u^3$

⑤ $y = x^2 \quad y = x^2$



SHELLS:

$x^2 = \sqrt{x}$

$x^4 = x$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$x = 0, \ x = 1$

$V = 2\pi \int_0^1 [x(\sqrt{x} - x^2)]\,dx$

$= 2\pi \int_0^1 (x^{\frac{3}{2}} - x^3)\,dx$

$= 2\pi \left(\frac{2}{5}x^{\frac{5}{2}} - \frac{1}{4}x^4\right)\Big|_0^1$

$= 2\pi\left(\frac{2}{5} - \frac{1}{4}\right) = \frac{3}{20}(2\pi) = \frac{3\pi}{10}\ u^3$