Math 152-AN Hour Exam #3 Satake

Name: KEY

Read all directions and problems *carefully*! Show all appropriate work for credit.

1. Find the derivative of the following functions. (18 pts.)

$y=\csc^{-1}(9x)$

$$\frac{dy}{dx} = \frac{-1}{|9x|\sqrt{(9x)^2-1}}(9)$$

$$= \boxed{\frac{-1}{|x|\sqrt{81x^2-1}}}$$

+6

$y=\sin^{-1}(9-x)-\tan^{-1}\sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(9-x)^2}}(-1) - \frac{1}{1+(\sqrt{x})^2}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$= \boxed{\frac{-1}{\sqrt{1-(9-x)^2}} - \frac{1}{2\sqrt{x}(1+x)}}$$

or

$$= \frac{-1}{\sqrt{-80+18x-x^2}} - \frac{1}{2\sqrt{x}(1+x)}$$

+12

2. Evaluate the following integrals involving inverse trigonometric functions. (20 pts.)

$$\int \frac{1}{25+4x^2}\,dx = \int \frac{1}{5^2+(2x)^2}\,dx$$

$u=2x$

$\frac{du}{2} = dx$

$$= \frac{1}{2}\int \frac{1}{5^2+u^2}\,du$$

$$= \frac{1}{2}\left(\frac{1}{5}\tan^{-1}\frac{u}{5}\right) + C$$

$$= \boxed{\frac{1}{10}\tan^{-1}\frac{2x}{5} + C}$$

+10

$$\int \frac{1}{\sqrt{4-(x+2)^2}}\,dx = \boxed{\sin^{-1}\frac{x+2}{2} + C}$$

+10

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3. Evaluate the following integral. (Complete the square) (9 pts.)

$$\int \frac{1}{(x-1)\sqrt{x^2-2x+1-1}}\,dx = \int \frac{1}{(x-1)\sqrt{(x-1)^2-1}}\,dx$$

$$= \frac{1}{1}\sec^{-1}\left|\frac{x-1}{1}\right| + C$$

$$= \boxed{\sec^{-1}|x-1| + C}$$

+9

4. Evaluate the following limits by first stating the type of indeterminate form and then applying L'Hopitals Rule. (10 pts.)

$\lim_{x\to\infty} x^{\frac{1}{x}}$; IF: ∞^0 LET $L = \lim_{x\to\infty} x^{\frac{1}{x}}$

$\ln L = \lim_{x\to\infty}\left[\frac{1}{x}\ln x\right]$

$\ln L = \lim_{x\to\infty}\frac{\ln x}{x}$; IF: $\frac{\infty}{\infty} \Rightarrow \ln L = \lim_{x\to\infty}\frac{\frac{1}{x}}{1} = 0$

$\Rightarrow L = e^0 = \boxed{1}$

+6

$\lim_{x\to 0^+} x^2 \cot x$; IF: $0\cdot\infty$

$\lim_{x\to 0^+}\frac{x^2}{\tan x}$; IF: $\frac{0}{0} \Rightarrow \lim_{x\to 0^+}\frac{2x}{\sec^2 x}$

$= \frac{0}{1} = \boxed{0}$

+4

5. Evaluate the integrals using a substitution to reduce it to standard form. (9 pts.)

$\int \frac{x^2+x+1}{x+1}\,dx$; long divide

$$\begin{array}{r} x \\ x+1\,\overline{)\,x^2+x+1} \\ -x^2-x\quad\downarrow \\ \hline 0\quad 0+1 \end{array}$$

$x + \frac{1}{x+1}$

$$\int\left(x+\frac{1}{x+1}\right)dx$$

$$= \boxed{\frac{x^2}{2} + \ln|x+1| + C}$$

+9

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$\int u\,dv = uv - \int v\,du$

6. Use Integration by Parts to evaluate the following integrals. (16 pts.)

$\int x e^{-2x}\,dx$

$u = x \qquad du = dx$

$v = -\frac{1}{2}e^{-2x} \qquad dv = e^{-2x}dv$

$= x\left(-\frac{1}{2}e^{-2x}\right) - \int -\frac{1}{2}e^{-2x}\,dx$

$= -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$

$= -\frac{1}{2}xe^{-2x} + \frac{1}{2}\left(-\frac{1}{2}e^{-2x}\right) + C$

$= \boxed{-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C}$

+8

$\int x^2 \ln x\,dx$

$u = \ln x \qquad du = \frac{1}{x}\,dx$

$v = \frac{x^3}{3} \qquad dv = x^2 dx$

$= \frac{1}{3}x^3\ln x - \int\left(\frac{x^3}{3}\right)\frac{1}{x}\,dx$

$= \frac{1}{3}x^3\ln x - \frac{1}{3}\int x^2\,dx$

$= \frac{1}{3}x^3\ln x - \frac{1}{3}\left(\frac{x^3}{3}\right) + C$

$= \boxed{\frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C}$

+8

7. Evaluate the following integral using Tic Tac Toe. (8 pts.)

$\int x^3 \cos x\,dx$

$\frac{dy}{dx}$	$\int dx$
$+ \to x^3$	$\cos x$
$- \to 3x^2$	$\sin x$
$+ \to 6x$	$-\cos x$
$- \to 6$	$-\sin x$
	$\cos x$

$\int x^3\cos x\,dx = \boxed{x^3\sin x + 3x^2\cos x - 6x\sin x - 6\cos x + C}$

+8

8. Integrate the following. (10 pts.)

$\int(\cot 2x + \sec x)\,dx$

$\int \cot 2x\,dx + \int \sec x\,dx$

$= \boxed{\frac{1}{2}\ln|\sin 2x| + \ln|\sec x + \tan x| + C}$

+10

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