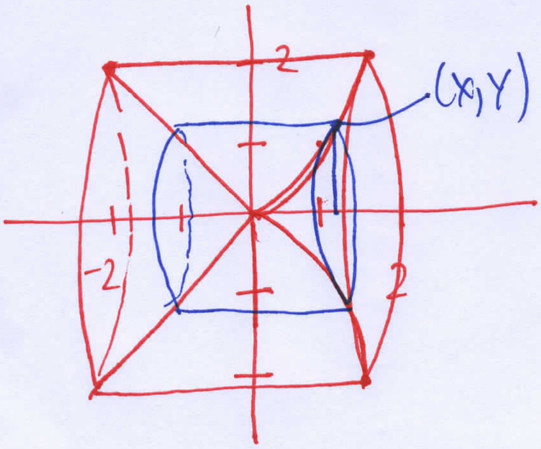
Math 152-AN   Hour Exam #2   Satake

Name: KEY

Read all directions and problems *carefully*! Show all appropriate work for credit.
Set up the integrals for Problems #1 and #2, but DO NOT Evaluate.

1. Use the Shell Method to find the volume of the solid that is generated by revolving the region bounded by the curves and lines $x=\sqrt{y}$, $x=-y$, and $y=2$ about the ***x*-axis**. (7 pts.)



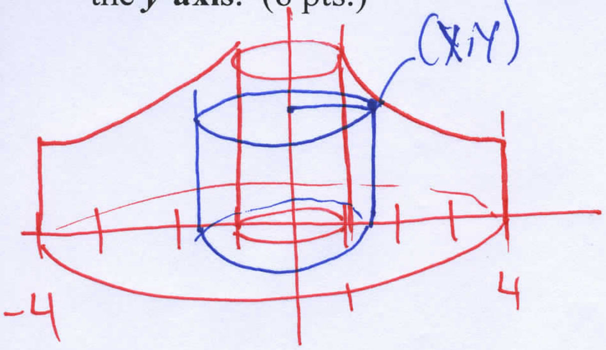
$y=x^2$ $(x\geq 0)$ $y=-x$

$$V=2\pi\int_0^2 y\left(\sqrt{y}-(-y)\right)dy$$

$$=\boxed{2\pi\int_0^2 y\left(\sqrt{y}+y\right)dy}$$

+7

2. Use the Shell Method to find the volume of the solid generated by revolving the region bounded by the following curves and lines $y=\frac{3}{2\sqrt{x}}$, $y=0$, $x=1$, and $x=4$ about the ***y*-axis**. (8 pts.)



$$V=2\pi\int_1^4 x\left(\frac{3}{2\sqrt{x}}-0\right)dx$$

$$=\boxed{2\pi\int_1^4 x\left(\frac{3}{2\sqrt{x}}\right)dx}$$

+8

15/15

3. Find the exact length of the curve $y = \frac{(x^2+2)^{\frac{3}{2}}}{3}$ from $x = 0$ to $x = 3$ by **setting up and evaluating the integral.** (7 pts.)

$$\frac{dy}{dx} = \frac{1}{\cancel{3}}\left[\frac{\cancel{3}}{\cancel{2}}(x^2+2)^{\frac{1}{2}}(\cancel{2}x)\right]$$

$$= x\sqrt{x^2+2}$$

$$\left(\frac{dy}{dx}\right)^2 = x^2(x^2+2)$$

$$L = \int_0^3 \sqrt{1 + x^4 + 2x^2}\, dx$$

$$= \int_0^3 \sqrt{(x^2+1)^2}\, dx$$

$$= \int_0^3 (x^2+1)\, dx$$

$$= \left(\frac{x^3}{3} + x\right|_0^3 = \frac{27}{3} + 3 - 0 = \boxed{12u}$$

+7

4. If a spring requires a force of 50 Newtons (N) to be stretched 0.4 m from its equilibrium position, calculate the work required to stretch a spring 0.5 m from its equilibrium position. Assume Hooke's law is obeyed. Set up the integral, but **do not evaluate**. (7 pts.)

$F = kx$

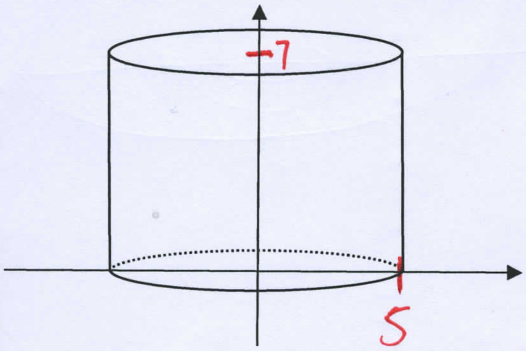
$50 = k(0.4)$

$k = 125$

$$\boxed{W = \int_0^{.5} 125x\, dx}$$

+7

5. A cylindrical tank has a height of 7 m and a radius of 5 m. If the tank is full of water, how much work is require to pump the water to the level of the top of the tank and out of the tank. Use 1000 kg/m^3 for the density of water and 9.8 m/s^2 for the acceleration due to gravity. Set up the integral and **approximate your answer using your calculator.** (7 pts.)



$A(y) = \pi(5)^2$

$$W = 1000(9.8)\int_0^7 (7-y)(25\pi)\, dy$$

$$= 245{,}000\pi \int_0^7 (7-y)\, dy$$

$$= \boxed{18{,}857{,}410 \text{ J}}$$

+7

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6. Evaluate the following integrals. (7 pts.)

$\int \frac{3x^3}{9-2x^4}\,dx$

$u = 9-2x^4$

$\frac{du}{-8} = \frac{\cancel{-8}x^3dx}{\cancel{-8}}$

$= -\frac{3}{8}\int \frac{1}{u}\,du$

$= \boxed{-\frac{3}{8}\ln|9-2x^4| + C}$

+3

$\int_1^{e^3} \frac{(\ln x)^2}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$u = \ln 1 = 0$

$u = \ln e^3 = 3$

$= \int_0^3 u^2\,du$

$= \frac{u^3}{3}\Big|_0^3$

$= \frac{3^3}{3} - 0 = \boxed{9u^2}$

+4

7. Evaluate the following integrals. (7 pts.)

$\int e^{\cos x}\sin x\,dx$

$u = \cos x$

$-du = +\sin x\,dx$

$= -\int e^u\,du$

$= -e^u + C$

$= \boxed{-e^{\cos x} + C}$

+3

$\int_1^9 \frac{3^{\sqrt{x}}}{\sqrt{x}}\,dx$

$u = x^{\frac{1}{2}}$

$du = \frac{1}{2}x^{-\frac{1}{2}}\,dx$

$2du = \frac{1}{\sqrt{x}}\,dx$

$u = 1^{\frac{1}{2}} = 1$

$u = 9^{\frac{1}{2}} = 3$

$= 2\int_1^3 3^u\,du$

$= 2\cdot\frac{3^u}{\ln 3}\Big|_1^3$

$= \frac{2}{\ln 3}(3^3 - 3)$

$= \boxed{\frac{48}{\ln 3}u^2}$

+4

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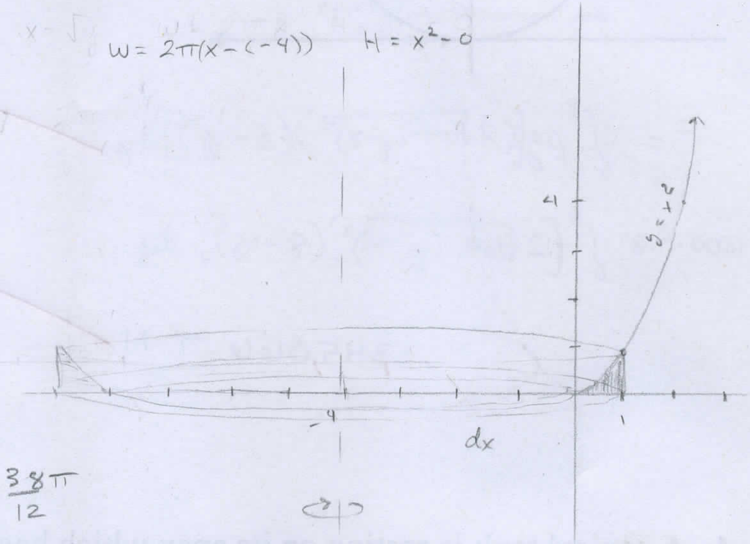
Math 152-AN Take Home Exa

Name:

You may not work with anyone else or seek help from a tutor or another instructor. You may use your text, homework, and notes. Take Home Exam due <u>at the beginning</u> of class on Monday, May 11th. Read all directions and problems *<u>carefully</u>*! Show all appropriate work for credit.

1. Let R be the region bounded by $y = x^2$, $x = 1$, and $y = 0$. Use the shell method to find the volume of the solid generated when R is revolved about the line $x = -4$. (25 pts.)

$W = 2\pi(x-(-4))$ $H = x^2 - 0$



$V = 2\pi \int_0^1 [(x+4)(x^2)]\,dx$

$= 2\pi \int_0^1 (x^3 + 4x^2)\,dx$

$= 2\pi \left(\frac{1}{4}x^4 + \frac{4}{3}x^3\right)\Big|_0^1$

$= 2\pi\left(\frac{1}{4} + \frac{4}{3}\right) = 2\pi\left(\frac{19}{12}\right) = \frac{38\pi}{12}$

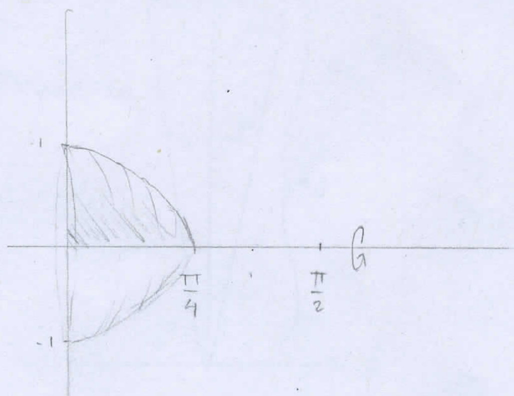
$= \frac{19\pi}{6} u^3$

2. Set up an integral for the area of the surface generated by revolving the curve $y = \cos x$, $0 \le x \le \frac{\pi}{4}$ about the x-axis. Approximate your answer using your calculator. (25 pts.)

$SA = 2\pi \int_0^{\frac{\pi}{4}} \cos x \left(\sqrt{1+(-\sin x)^2}\right) dx$

$\frac{dy}{dx} = -\sin x$

$\approx 4.79\, u^2$



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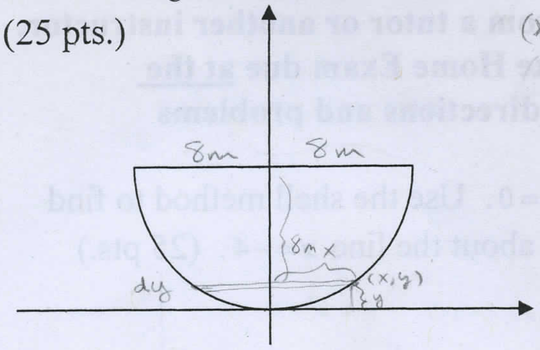
3. A dam is in the shape of a semicircle and it measures 16 m across its top. Assuming that the water level is at the top of the dam, find the total force on the face of the dam. Use 1000 kg/m^3 for the density of water and 9.8 m/s^2 for the acceleration due to gravity. (25 pts.)

$(x-h)^2+(y-k)^2=r^2$

$x^2+(y-8)^2=64$

$(y-8)^2=64-x^2 \qquad y-8=\pm\sqrt{64-x^2}$

$y=\pm\sqrt{64-x^2}+8$



$w=2x=2\sqrt{64-(y-8)^2}$

$D=8-y$

$y=-\sqrt{64-x^2}+8$

$y-8=-\sqrt{64-x^2}$

$(y-8)^2=64-x^2$

$(y-8)^2-64=-x^2$

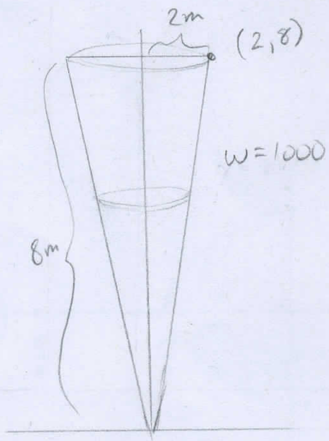
$\sqrt{64-(y-8)^2}=x$

$$F=\int_0^8 \rho g\left[\left(2\sqrt{64-(y-8)^2}\right)(8-y)\right]dy$$

$$=1000\cdot 9.8\int_0^8\left[2\sqrt{64-(y-8)^2}\,(8-y)\right]dy$$

$$=3,345,066.7\text{ N}$$

4. A conical tank is resting on its apex which has a height of 8 m and a base radius of 2 m. If the tank is full, how much work is required to pump the water to the level of the top of the tank and out of the tank? Use 1000 kg/m^3 for the density of water and 9.8 m/s^2 for the acceleration due to gravity. (25 pts.)



$m=4 \qquad y=4x \qquad x=\frac{y}{4}$

$A=\pi(x)^2=\pi\left(\frac{y}{4}\right)^2=\frac{\pi y^2}{16}$

$$w=1000\cdot 9.8\cdot\frac{\pi}{16}=\int_0^8 y^2(8-y)\,dy$$

$$=\frac{9800\pi}{16}\int_0^8(8y^2-y^3)\,dy$$

$$=\frac{9800\pi}{16}\left(\frac{8}{3}y^3-\frac{1}{4}y^4\right)\Big|_0^8$$

$$=\frac{9800\pi}{16}\left(\frac{4096}{3}-1024\right)$$

$$=\frac{9800\pi}{\cancel{16}_{1}}\left(\frac{\cancel{1024}^{64}}{3}\right)=\frac{627200\pi}{3}\text{ J}$$

≈ 656802 J

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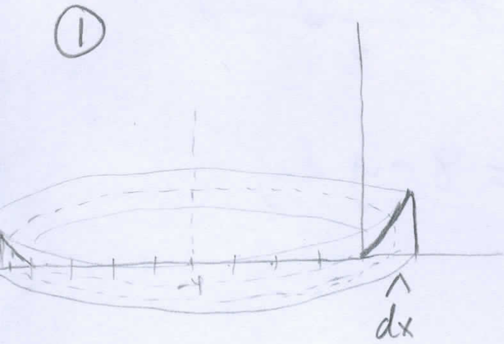
Take Home Exam #2

①



$$V = 2\pi \int_0^1 \left[(x+4)(x^2)\right] dx$$

$$= 2\pi \int_0^1 (x^3+4x^2)\, dx$$

$$= 2\pi \left(\frac{x^4}{4} + \frac{4x^3}{3}\right)\Big|_0^1$$

$$= 2\pi\left(\frac{1}{4}+\frac{4}{3}\right)$$

$$= 2\pi\left(\frac{3+16}{\cancel{12}\,6}\right)$$

$$\boxed{= \frac{19\pi}{6}\,u^3}$$

②

$$\frac{dy}{dx} = -\sin x \qquad (-\sin x)^2 = \sin^2 x$$

$$S = \int_0^{\pi/4} \left[2\pi \cos x \sqrt{1+\sin^2 x}\right] dx$$

$$= 2\pi \int_0^{\pi/4} \left[\cos x \sqrt{1+\sin^2 x}\right] dx$$

$$\approx 2\pi(.76225218) \approx 4.789371698$$

$$\boxed{\approx 4.79\, u^2}$$

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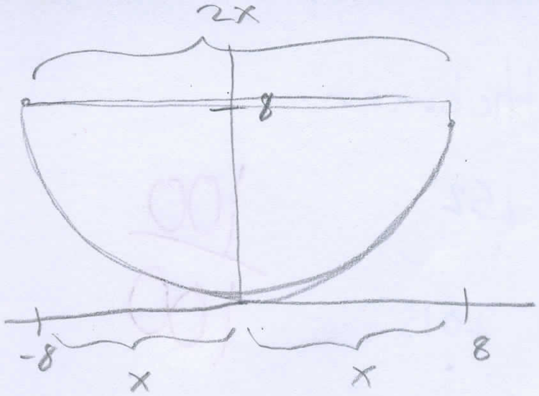
③ $x^2+(y-8)^2=64$

$x^2=\cancel{64}-y^2+16y-\cancel{64}$

$x=\pm\sqrt{16y-y^2}$



$F=\rho g\int_0^8\left[(8-y)\,2\sqrt{16y-y^2}\right]dy$ let $u=16y-y^2$

$\frac{du}{2}=\frac{\cancel{16}-\cancel{2}y\,dy}{2}$

$F=\frac{\cancel{2}\rho g}{\cancel{2}}\int_{y=0}^{y=8}\left[u^{\frac{1}{2}}\right]du$ ← $\frac{1}{2}du=8-y\,dy$

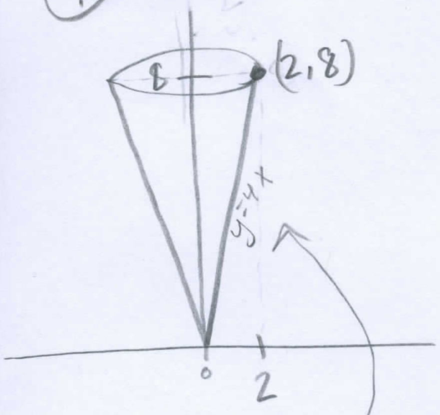
$F=9800\int_{u=0}^{u=64}(u^{\frac{1}{2}})\,du=9800\left(\frac{2}{3}u^{\frac{3}{2}}\right|_0^{64}=9800\left(\frac{1024}{3}\right)$

$\boxed{=3{,}345{,}066.667\ \text{N}}$

$\approx 3.345\times10^7$

25

4



$\frac{8-0}{2-0} = 4$

$y = 4x + 0$

$\frac{1}{4}y = x$

$$w = \rho g \int_0^8 \left[\pi \left(\tfrac{1}{4}y\right)^2 (8-y)\right] dy$$

$$= \frac{\rho g \pi}{16} \int_0^8 \left(y^2(8-y)\right) dy$$

$$= \frac{9800\pi}{16} \int_0^8 (8y^2 - y^3)\, dy$$

$$= \frac{1225\pi}{2} \left(\frac{8y^3}{3} - \frac{y^4}{4}\right)\Big|_0^8$$

$$= \frac{1225\pi}{2}\left(\frac{4096}{3} - \frac{4096}{4}\right) = \frac{1225\pi}{2}\left(\frac{4096}{12}\right)$$

$$= \frac{1225\pi}{\cancel{2}}\left(\frac{\cancel{1024}}{3}\right) = 1225\pi\left(\frac{512}{3}\right)$$

$$\boxed{= \frac{627{,}200\,\pi}{3}\ \text{J}}$$

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