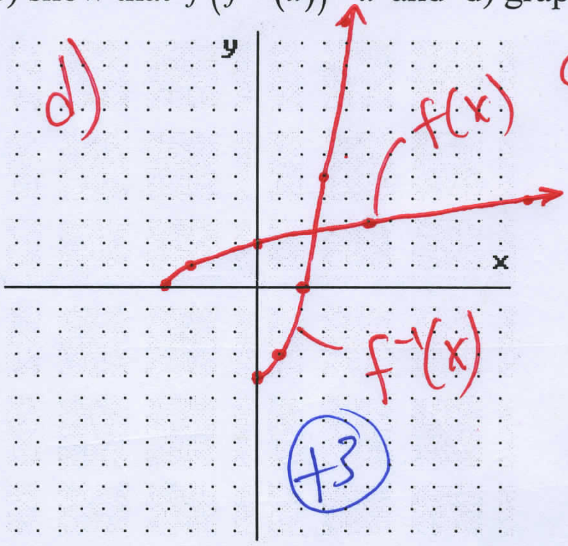
Math 152-AN   Hour Exam #1   Satake

Name: KEY

Read all directions and problems *carefully*! Show all appropriate work for credit.

1. For the function $f(x)=\sqrt{x+4}$, a) find $f^{-1}(x)$ b) identify the Domain and Range of $f^{-1}(x)$ c) show that $f\left(f^{-1}(x)\right)=x$ and d) graph $f(x)$ and $f^{-1}(x)$ on the same set of axes. (10 pts.)

d)



a) $y=\sqrt{x+4}$

$x^2=\left(\sqrt{y+4}\right)^2$

$x^2=y+4$

$\underline{-4 \qquad -4}$

$\boxed{f^{-1}(x)=x^2-4}$ (+3)

b) $D_{f^{-1}}: [0,\infty)$

$R_{f^{-1}}: [-4,\infty)$ (+2)

c) $f\circ f^{-1}=\sqrt{x^2-4+4}$

$=\sqrt{x^2}=|x|$

$=\boxed{x}$ SINCE $x\geq 0$. (+2)

2. Find the derivative for the following functions. (20 pts.)

$y=\dfrac{\ln 5x}{e^{2x}}$

$$\frac{dy}{dx}=\frac{\frac{1}{\cancel{5}x}(\cancel{5})\cdot e^{2x}-\ln 5x\cdot e^{2x}(2)}{\left(e^{2x}\right)^2}$$

$$=\boxed{\frac{e^{2x}\left[\frac{1}{x}-2\ln 5x\right]}{e^{4x}}}$$ (+7)

$y=-x^3\ln(\sin x)$

$$\frac{dy}{dx}=-3x^2\cdot\ln(\sin x)+(-x^3)\frac{1}{\sin x}\left(\frac{\cos x}{1}\right)$$

$$=\boxed{-3x^2\ln(\sin x)-x^3\cot x}$$ (+7)

$y=\log_4\left(4x^2-64\right)$

$$\frac{dy}{dx}=\frac{1}{4x^2-64}\cdot\frac{1}{\ln 4}(8x)$$

$$=\frac{\overset{2}{\cancel{8}}x}{\cancel{4}\ln 4(x^2-16)}=\boxed{\frac{2x}{(x^2-16)\ln 4}}$$ (+6)

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3. Use logarithmic differentiation to find the derivative of y. (8 pts.)

$$y=\frac{\left(x^2+1\right)^2}{\sqrt[3]{x+8}} \Rightarrow \ln y = \ln\left(x^2+1\right)^2 - \ln(x+8)^{\frac{1}{3}} = 2\ln(x^2+1) - \frac{1}{3}\ln(x+8)$$

$$\frac{1}{y}\frac{dy}{dx} = 2\frac{1}{x^2+1}(2x) - \frac{1}{3}\frac{1}{x+8} = \left[\frac{4x}{x^2+1} - \frac{1}{3(x+8)}\right]\cdot y$$

+8

$$\boxed{\frac{dy}{dx} = \left[\frac{4x}{x^2+1} - \frac{1}{3(x+8)}\right]\frac{(x^2+1)^2}{\sqrt[3]{x+8}}}$$

4. Find the derivative for the following functions. (20 pts.)

$y=e^{-\frac{1}{x^2}}+x\cdot 2^x$

$$\frac{dy}{dx} = e^{-\frac{1}{x^2}}\left(\frac{2}{x^3}\right) + 1\cdot 2^x + x\cdot 2^x \ln 2$$

$$= \boxed{\frac{2e^{-\frac{1}{x^2}}}{x^3} + 2^x + x\cdot 2^x\cdot \ln 2}$$

+7

$y=9^{\log x}$

$$\frac{dy}{dx} = 9^{\log x}\cdot \ln 9\cdot \frac{1}{x}\cdot\frac{1}{\ln 10}$$

$$= \boxed{\frac{9^{\log x}\cdot \ln 9}{x\cdot \ln 10}}$$

+6

$y=\frac{2}{e^x-e^{-x}} = 2\left[e^x-e^{-x}\right]^{-1}$

$$\frac{dy}{dx} = 2\left(-1\left[e^x-e^{-x}\right]^{-2}\left(e^x-e^{-x}(-1)\right)\right)$$

$$= \boxed{\frac{-2(e^x+e^{-x})}{\left[e^x-e^{-x}\right]^2}}$$

OR

+7

5. A projectile is launched vertically from the ground at $t=0$ and its velocity in flight (in m/s) is given by $v(t)=20-10t$. Find the position function, the displacement, and total distance traveled after t seconds for $0 \le t \le 4$, assuming $s(0)=0$. (8 pts.)

$$s(t) = s(0) + \int(20-10t)\,dt$$

$$s(t) = 0 + 20t - 10\left(\frac{t^2}{2}\right) = \boxed{-5t^2+20t}$$

+8

DISPL:

$$\int_0^4 (20-10t)\,dt = \left(20t-5t^2\right|_0^4 = \boxed{0\text{m}}$$

DIST:

$$\int_0^4 |20-10t|\,dt = \int_0^2 (20-10t)\,dt + \int_2^4 \left[-1(20-10t)\right]dt = 20-0+\left[0-(-20)\right] = \boxed{40\text{m}}$$

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6. Given that the population of the earth was 6.0 billion people in 1999 ($t = 0$) and 6.9 billion people in 2009 ($t = 10$), find: a) an exponential growth function for the world's population that fits that two data points. b) Find the doubling time for the world population using the model in part a). c) Find the instantaneous growth rate of the population in 2014 ($t = 15$). (8 pts.)

LET $P(t)$ = WORLD POPULATION t YEARS AFTER 1999 IN BILLIONS

a) $P(t) = P_0 e^{kt} \Rightarrow P(10) = 6.0e^{k \cdot 10}$ MUST EQUAL 6.9

$6.0e^{10k} = 6.9$

$\ln e^{10k} = \ln \frac{6.9}{6.0}$

$k = \frac{\ln\left(\frac{6.9}{6}\right)}{10} \approx 0.01398$

$\boxed{P(t) = 6.0e^{0.014t}}$

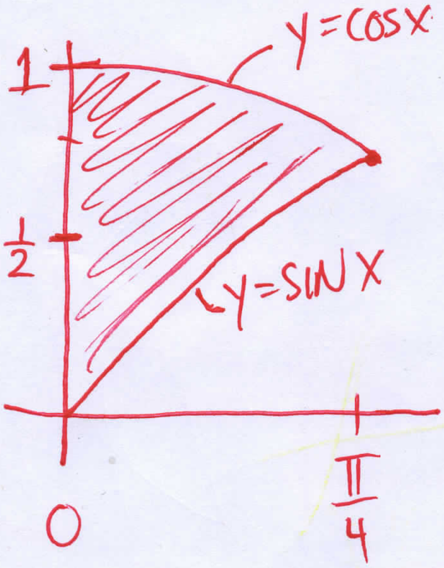
b) $T_2 = \frac{\ln 2}{0.014} \approx \boxed{49.5 \text{ YEARS}}$

c) $P'(t) = 6.0e^{0.014t}(0.014)$

$P'(15) = 6.0(0.014)e^{0.014(15)} \approx \boxed{0.104 \frac{\text{BILL}}{\text{YR}}}$

+8

Sketch the region bounded by $y = \cos x$, $y = \sin x$, $x = 0$ and finds its area. (10 pts.)



$A = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx$

$= \left(\sin x + \cos x\right|_0^{\frac{\pi}{4}}$

$= \sin\frac{\pi}{4} + \cos\frac{\pi}{4} - \sin 0 - \cos 0$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 0 - 1$

$= \boxed{(\sqrt{2} - 1)\, u^2}$

+10

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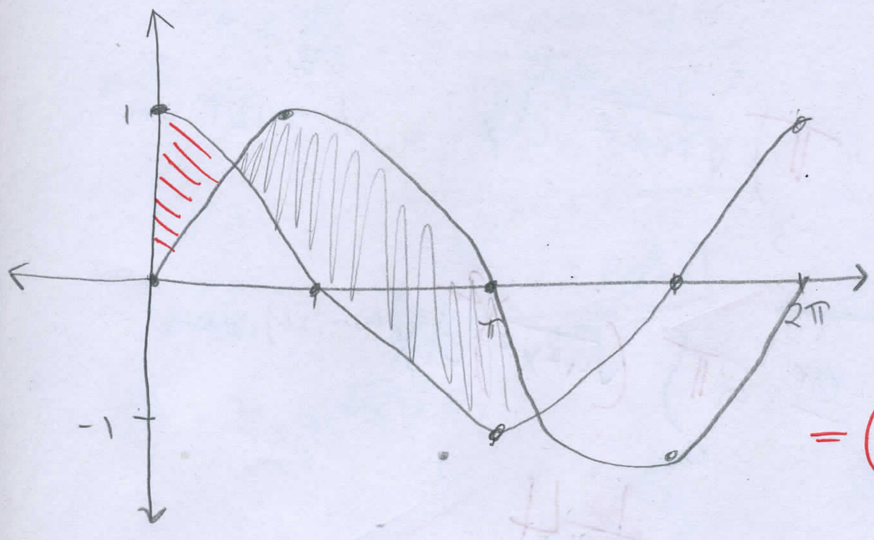
7. Sketch the region bounded by $y = \cos x$, $y = \sin x$, $x = 0$ and finds its area. (10 pts.)



$\cos x = \sin x$

$\cot x = 1$

$x = \tan^{-1}(1)$

$x = \pi/4 \, , 5\pi/4$

$$\int_{\pi/4}^{5\pi/4} |\sin x - \cos x| \, dx$$

$$= \left(-\cos x - \sin x \right|_{\pi/4}^{5\pi/4}$$

$$= -\cos \frac{5\pi}{4} - \sin \frac{5\pi}{4} + \left(+\cos(\pi/4) + \sin(\pi/4) \right)$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$$

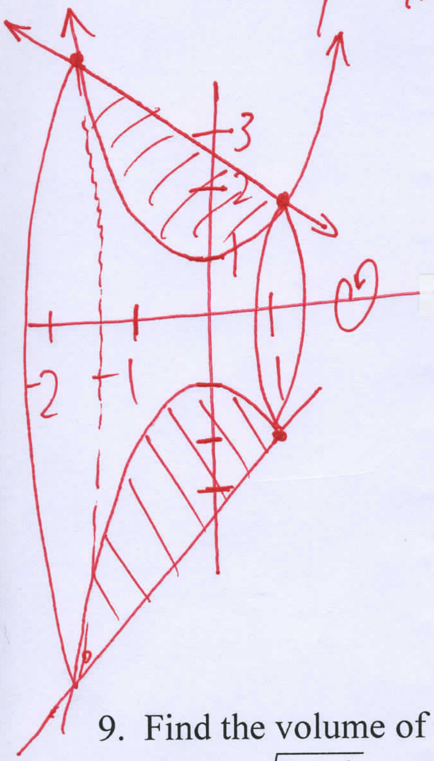
$$= \frac{4\sqrt{2}}{2} = \boxed{2\sqrt{2} \text{ un}^2}$$

Set up the integrals for the following, but <u>DO NOT SOLVE</u>.

8. Find the volume of the solid generated by revolving the region bounded by the given curves about the ***x*-axis**. (8 pts.)

$y = x^2 + 1,\ x + y = 3$

$y = -x + 3$



$$V = \pi \int_a^b \left[R(x)^2 - r(x)^2 \right] dx$$

$x^2 + 1 = -x + 3$

$x^2 + x - 2 = 0$

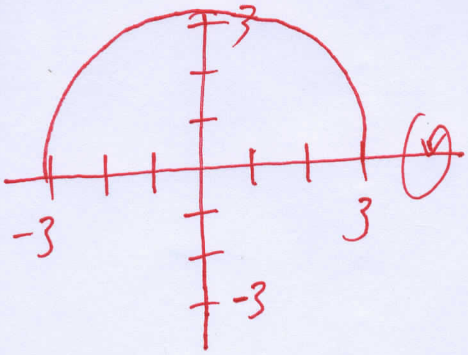
$(x+2)(x-1) = 0$

$x = -2;\ x = 1$

$$= \boxed{\pi \int_{-2}^{1} \left[(-x+3)^2 - (x^2+1)^2 \right] dx}$$

+8

9. Find the volume of the solid that is generated by revolving the region bound by the curves $y = \sqrt{9 - x^2}$ and $y = 0$ about the ***x*-axis**. (8 pts.)



$$V = \pi \int_a^b [\text{RADIUS}]^2 dx$$

$$= \pi \int_{-3}^{3} \left(\sqrt{9 - x^2} \right)^2 dx$$

$$= \boxed{\pi \int_{-3}^{3} (9 - x^2)\, dx}$$

+8

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