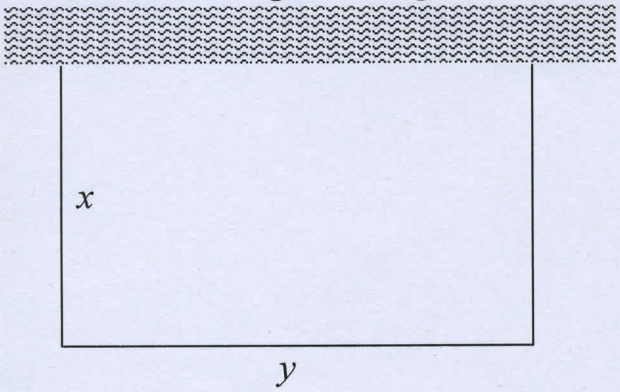
Math 148-AN Hour Exam #4 Satake

Name: KEY

Read all directions and problems carefully! Show all appropriate work for credit.
Answer one or both of Problems #1 and #2. (25 pts.)

1. A Canadian farmer has 4000 meters of fence to enclose a rectangular plot that borders on a river. If the farmer does not fence the side along the river, what is the largest area for constructing a rectangular corral? **Set up the function**, but do not maximize the area.



PRIMARY EQN:

AREA = LENGTH(WIDTH)

$A = y(x)$

$\boxed{A(x) = (4000-2x)x}$

OR

$\boxed{A(x) = 4000x - 2x^2}$

SECONDARY EQN:

$2x + y = 4000\text{ m}$

$\Rightarrow y = 4000 - 2x$

2. The price p (in dollars) and the quantity q (in units) sold of a certain product obey the demand equation , $p = -\frac{1}{6}q + 100$. **Use your calculator** to answer the following:

a) What quantity q maximizes revenue? b) What is the maximum revenue? c) What price should the company charge to maximize revenue?

REVENUE $= q \cdot p \Rightarrow R(q) = q\left(-\frac{1}{6}q + 100\right)$

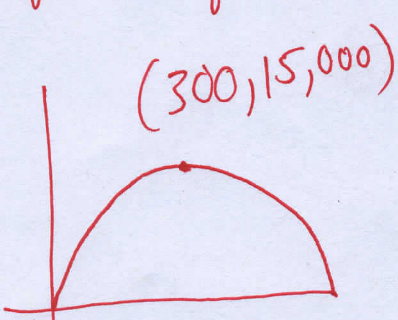
a) $R(q) = -\frac{1}{6}q^2 + 100q \Rightarrow$ 2ND CALC MAX

$\boxed{300 \text{ UNITS}}$

WINDOW:

X: [0, 600, 100]

Y: [0, 25,000, 5000]



b) $\boxed{\$15{,}000}$

c) $p = -\frac{1}{6}(300) + 100$

$\boxed{p = \$50}$

25

Answer one or both of Problems #3 and #4. (25 pts.)

3. A spherical balloon is being inflated by a pump at the rate of 2 cubic inches per second. How fast is the diameter changing when the radius is 0.5 inches? [Volume of a sphere, $V = \frac{4}{3}\pi r^3$]

LET V = VOLUME
" r = RADIUS
" t = TIME
" D = DIAMETER

GOAL:

$$\left.\frac{dr}{dt}\right|_{r=0.5\text{ IN}}$$

GIVEN:

$$\frac{dV}{dt} = 2\frac{\text{IN}^3}{\text{SEC}}$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi(\cancel{3}r^2)\frac{dr}{dt}$$

$$\frac{\frac{dV}{dt}}{4\pi r^2} = \frac{\cancel{4\pi r^2}\cdot\frac{dr}{dt}}{\cancel{4\pi r^2}}$$

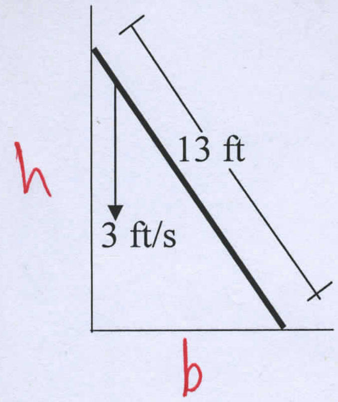
$$\frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$$

$$\left.\frac{dr}{dt}\right|_{r=0.5} = \frac{1}{4\pi(.5)^2}(2)$$

$$\frac{dr}{dt} = \frac{2}{\pi}\frac{\text{IN}}{\text{SEC}}$$

$$\frac{dD}{dt} = 2\left(\frac{2}{\pi}\right) = \boxed{\frac{4}{\pi}\frac{\text{IN}}{\text{SEC}}}$$

4. A ladder 13 feet in length rests against a vertical wall and is sliding down the wall at the rate of 3 feet per second. How fast is the foot of the ladder moving away from the wall when it is 12 feet from the base of the wall?



LET h = HEIGHT
" b = BASE
" t = TIME

GIVEN:

$$\frac{dh}{dt} = -3\,\text{FT/SEC}$$

GOAL:

$$\left.\frac{db}{dt}\right|_{b=12\text{ FT}}$$

$$h^2 + b^2 = 13^2$$

$$2h\left(\frac{dh}{dt}\right) + 2b\left(\frac{db}{dt}\right) = 0$$

$$\frac{+\cancel{2b}\frac{db}{dt}}{\cancel{2b}} = \frac{-\cancel{2}h\left(\frac{dh}{dt}\right)}{\cancel{2}b}$$

$$\left.\frac{db}{dt}\right|_{b=12} = \frac{-h}{b}\left(\frac{dh}{dt}\right)$$

$$= \frac{-5}{12}\left(\frac{-3}{1}\right)$$

$$\boxed{\left.\frac{db}{dt}\right|_{b=12} = \frac{5}{4}\frac{\text{FT}}{\text{SEC}}}$$

25

5. Find $\frac{dy}{dx}$ by implicit differentiation for the following equation. (25 pts.)

$x^2y - 5x = 3\sqrt{y} - e^3$

$2x(1)\cdot y + x^2\left(\frac{dy}{dx}\right) - 5(1) = 3\left(\frac{1}{2}y^{-\frac{1}{2}}\right)\left(\frac{dy}{dx}\right) - 0$

$2xy + x^2\left(\frac{dy}{dx}\right) - 5 = \frac{3}{2}y^{-\frac{1}{2}}\left(\frac{dy}{dx}\right)$

$\frac{2xy - 5}{[\quad]} = \frac{\left[\frac{3}{2}y^{-\frac{1}{2}} - x^2\right]\left(\frac{dy}{dx}\right)}{[\quad]}$

$\frac{dy}{dx} = \boxed{\frac{2xy-5}{\frac{3}{2}y^{-\frac{1}{2}} - x^2}} = \boxed{\frac{5-2xy}{x^2 - \frac{3}{2}y^{-\frac{1}{2}}}}$

+13

$\ln x + e^{x+y} = -x^{\frac{3}{4}}$

$\frac{1}{x} + e^{x+y}\left(1 + \frac{dy}{dx}\right) = -\frac{3}{4}x^{-\frac{1}{4}}(1)$

$e^{x+y} + e^{x+y}\left(\frac{dy}{dx}\right) = -\frac{3}{4}x^{-\frac{1}{4}} - \frac{1}{x}$

$\frac{\cancel{e^{x+y}}\left(\frac{dy}{dx}\right)}{\cancel{e^{x+y}}} = \frac{-\frac{3}{4}x^{-\frac{1}{4}} - \frac{1}{x} - e^{x+y}}{e^{x+y}}$

$\frac{dy}{dx} = \boxed{\frac{-\frac{3}{4}x^{-\frac{1}{4}} - \frac{1}{x} - e^{x+y}}{e^{x+y}}}$

+12

6. Evaluate the indefinite integrals. (25 pts.)

$\int\left(\frac{1}{14}x^7 - \frac{9}{\sqrt[3]{x}} + e^x + 8\right)dx$

$= \frac{1}{14}\int x^7dx - 9\int x^{-\frac{1}{3}}dx + \int e^x dx + \int 8dx$

$= \frac{1}{14}\frac{x^8}{8} - 9\left(\frac{x^{\frac{2}{3}}}{\frac{2}{3}}\right) + e^x + 8x + C$

$= \boxed{\frac{1}{112}x^8 - \frac{27}{2}x^{\frac{2}{3}} + e^x + 8x + C}$

+16

$\int \frac{x^8 - 4x^4}{2x^2}dx = \int\left(\frac{x^8}{2x^2} - \frac{4x^4}{2x^2}\right)dx$

$= \frac{1}{2}\int x^6dx - 2\int x^2dx$

$= \frac{1}{2}\left(\frac{x^7}{7}\right) - 2\left(\frac{x^3}{3}\right) + C$

$= \boxed{\frac{1}{14}x^7 - \frac{2}{3}x^3 + C}$

+9

50