Math 148-AN | Hour Exam #3 | Satake

Name: KEY

Read all directions and problems carefully! Show all appropriate work for credit.

1. For the function $f(x)=x^3-6x^2+9x$, find a) the x-intercepts; b) all critical points; c) the open intervals where the function is increasing; d) the open intervals where the function is decreasing; e) any relative extrema and write the **exact** value(s) of those relative extrema; f) describe explicitly how the First Derivative Test was used to determine a max or a min. (16 pts.)

a) $f(x)=x^3-6x^2+9x=0$

$x(x^2-6x+9)=0$

$x(x-3)(x-3)=0$

$x=0$; $x=3$

$\boxed{(0,0) ; (3,0)}$ +2

b) $f'(x)=3x^2-12x+9=0$

$3(x^2-4x+3)=0$

$3(x-1)(x-3)=0$

$x=1$; $x=3$

$f(1)=4$; $f(3)=0$

$\boxed{(1,4) ; (3,0)}$ +4

c) INCR: $\boxed{(-\infty,1)\cup(3,\infty)}$ +2

d) DECR: $\boxed{(1,3)}$ +2

e) REL. MAX OF 4 @ 1

REL. MIN OF 0 @ 3. +4

2. For the function from Problem #1, $f(x)=x^3-6x^2+9x$, find a) any inflection points; b) the open intervals when the function is concave upward; c) the open intervals when the function is concave downward; d) describe explicitly how the **Second** Derivative Test was used to confirm the extrema found in Problem #1. (16 pts.)

a) $f''(x)=6x-12=0$

$x=2$

$f(2)=2$

$\boxed{(2,2)}$ +4

b) CCU: $\boxed{(2,\infty)}$ +3

c) CCD: $\boxed{(-\infty,2)}$ +3

d) $f''(1)<0 \Rightarrow$ REL MAX.

$f''(3)>0 \Rightarrow$ REL MIN +6

1 2 3

f': ++++ | - - - - - - - - | ++++++

$\boxed{\#1 f)}$ +2 - CHANGE FROM INCR. TO DECR 'ACROSS' A CN $\Rightarrow$ REL. MAX.

- CHANGE FROM DEC. TO INCR 'ACROSS' A CN $\Rightarrow$ REL MIN.

f'': - - - - - - - - - | +++++++++++

32

3. Given the following function $f(x)=xe^x$ and its first and second derivative $f'(x)=e^x+xe^x$ and $f''(x)=2e^x+xe^x$, respectively. Determine a) the critical numbers; b) the inflection numbers; c) the open intervals on which $f(x)$ is increasing and decreasing; d) the open intervals on which $f(x)$ is concave up and concave down. (18 pts.)

a) $f'(x)=e^x(1+x)=0$; $f''(x)=e^x(2+x)$

CN's: $e^x \neq 0$; $1+x=0$
$x=-1$

IN's: $e^x \neq 0$; $2+x=0$
$x=-2$

-2 $\quad$ -1

f': ------ DECR ------ | ++++ INCR ++++

f'': ---CCD--- | ++++ CCU +++++

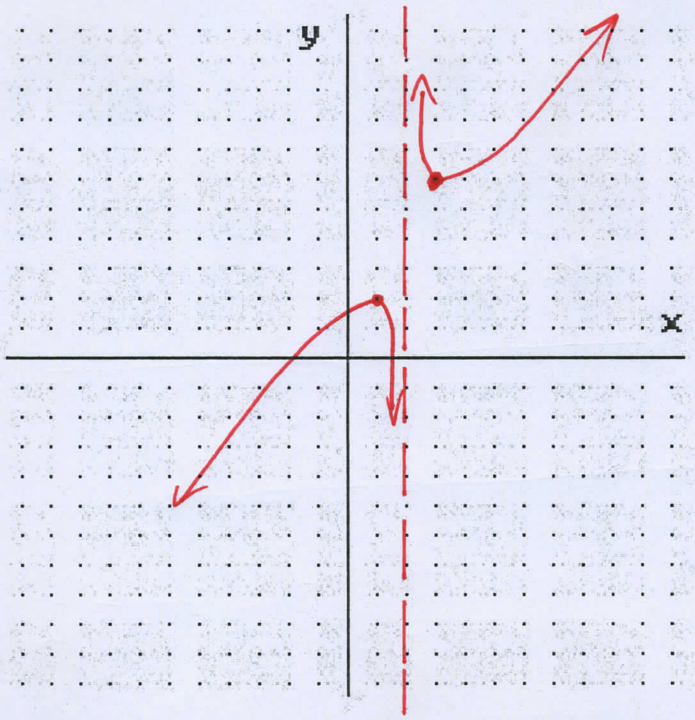
c) DECR: $(-\infty, -1)$
INCR: $(-1, \infty)$

d) CCD: $(-\infty, -2)$
CCU: $(-2, \infty)$

18

4. For the given function and its derivatives, $f(x) = \frac{x^2-3}{x-2}$ and $f'(x) = \frac{(x-3)(x-1)}{(x-2)^2}$,

$f''(x) = \frac{2}{(x-2)^3}$, write a) the domain; b) the critical numbers; c) the inflection numbers; d) the relative extrema; and e) draw a careful graph of $f(x)$. (18 pts.)



a) $(-\infty, 2) \cup (2, \infty)$ +4

b) CN'S: $x=1$; $x=2$; $x=3$ +6

c) IN: $x=2$ +2

d) RELATIVE MAX OF 2 @ 1.
RELATIVE MIN OF 6 @ 3. +6

5. For the function $f(x) = \ln(x^2+9)$, find the open intervals when the function is concave up and concave down. (16 pts.)

$f'(x) = \frac{1}{x^2+9}(2x)$

$f''(x) = \frac{2(x^2+9) - 2x(2x)}{(x^2+9)^2}$

$= \frac{2x^2+18-4x^2}{(\ )^2} = \frac{-2x^2+18}{(x^2+9)^2}$ $= 0$

$(x^2+9)^2 \rightarrow$ NEVER EQUALS 0.

$\rightarrow -2(x^2-9) = 0$

$-2(x-3)(x+3) = 0$

IN'S: $x=-3$; $x=3$

$$\boxed{\begin{array}{l} \text{CCD: } (-\infty, -3) \cup (3, \infty) \\ \text{CCU: } (-3, 3) \quad +16 \end{array}}$$

$-3 \qquad 3$

f'': - - CCD - - | ++++ CCU +++ | - - - CCD - - - -

34

6. Find the **absolute** extrema, as well as all values of x where they occur for $f(x) = x^4 - 4x^3$, on the domain $[1, 4]$. (16 pts.)

$f'(x) = 4x^3 - 12x^2 = 0 \Rightarrow 4x^2(x-3) = 0$

$\frac{4x^2}{4} = \frac{0}{4}$ or $x - 3 = 0$

$x^2 = 0$ $\qquad$ $x = 3$

$\sqrt{x^2} = \pm\sqrt{0}$

$x = 0$

CN'S: $x = 0$; NOT IN THE INTERVAL $[1, 4]$

	1	2	3	4
f'	- - - DECR - -		+ INCR +	

TEST THE CN + END POINTS:

$f(1) = (1)^4 - 4(1)^3 = -3$

$f(3) = (3)^4 - 4(3)^3 = -27$; MIN.

$f(4) = (4)^4 - 4(4)^3 = 0$; MAX.

$\boxed{\text{ABSOLUTE MAX OF } 0 \text{ @ } 4. \quad \text{ABSOLUTE MIN OF } -27 \text{ @ } 3.}$

+16

16