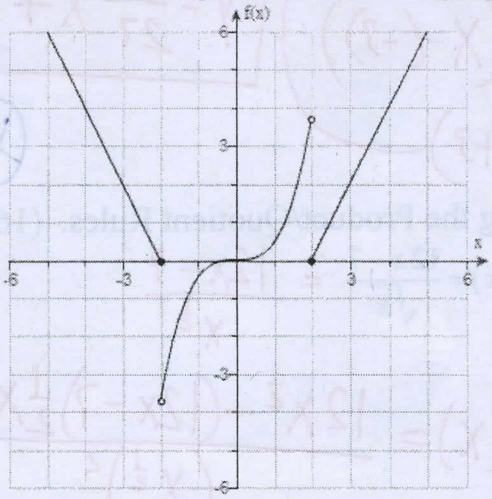
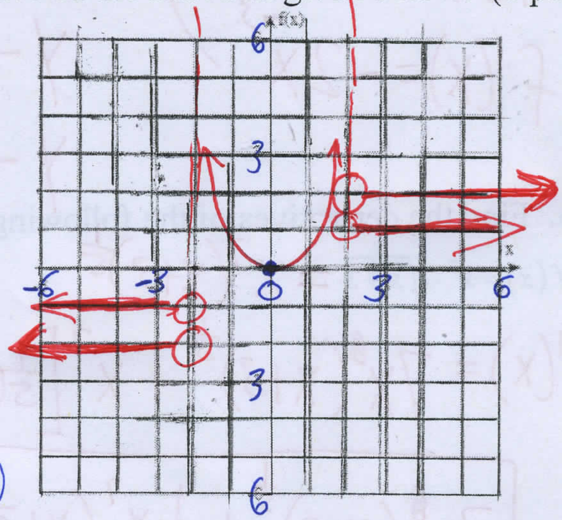
Math 148-AN　　　　Hour Exam #2　　　　Satake

Name: KEY

Read all directions and problems carefully! Show all appropriate work for credit.

1. Draw a rough sketch of the graph of the derivative of the function given below. (8 pts.)





+8

2. Find the derivative of the function defined as follows. (12 pts.)

$$y=-8x^4+\frac{10}{x^2}+12\sqrt[3]{x}-16x^{-\frac{3}{4}} = -8x^4+10x^{-2}+12x^{\frac{1}{3}}-16x^{-\frac{3}{4}}$$

$$\frac{dy}{dx}=-8(4x^3)+10(-2x^{-3})+12\left(\tfrac{1}{3}x^{-\frac{2}{3}}\right)-\overset{4}{\cancel{16}}\left(-\tfrac{3}{4}x^{-\frac{7}{4}}\right)$$

$$=\boxed{-32x^3-20x^{-3}+4x^{-\frac{2}{3}}+12x^{-\frac{7}{4}}}$$

+12

3. For the function $f(x)=2x^3+9x^2-60x+4$, find all values of x where the tangent line is horizontal. (8 pts.)

$$f'(x)=6x^2+18x-60=0$$

$$=6(x^2+3x-10)=0$$

$$6(x+5)(x-2)=0$$

$$\boxed{x=-5;\ x=2}$$

+8

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4. Find the equation for the tangent to the curve at the given point. (8 pts.)

$f(x)=\frac{1}{x^2}$; $\left(-3,\frac{1}{9}\right)$

$f(x)=x^{-2}$

$f'(x)=-2x^{-3}$

$f'(-3)=\frac{-2}{(-3)^3}=\frac{2}{27}$

$y-\left(\frac{1}{9}\right)=\frac{2}{27}(x-(-3))$

$y-\frac{1}{9}=\frac{2}{27}(x+3)$

$y=\frac{2}{27}x+\frac{2}{9}+\frac{1}{9}$

$\boxed{y=\frac{2}{27}x+\frac{1}{3}}$

+8

5. Find the derivatives of the following functions using the Product/Quotient Rules. (16 pts.)

$f(x)=x^7\cdot\sqrt[5]{x+3}=x^7(x+3)^{\frac{1}{5}}$

$f'(x)=7x^6(x+3)^{\frac{1}{5}}+x^7\left[\frac{1}{5}(x+3)^{-\frac{4}{5}}\right]$

$=\boxed{7x^6(x+3)^{\frac{1}{5}}+\frac{1}{5}x^7(x+3)^{-\frac{4}{5}}}$

+8

$f(x)=\frac{12x-7}{\sqrt{x}}=\frac{12x-7}{x^{\frac{1}{2}}}$

$f'(x)=\frac{12x^{\frac{1}{2}}-(12x-7)\frac{1}{2}x^{-\frac{1}{2}}}{\left(x^{\frac{1}{2}}\right)^2}$

$=\boxed{\frac{12x^{\frac{1}{2}}-\frac{1}{2}x^{-\frac{1}{2}}(12x-7)}{x}}$

+8

6. Use the Chain Rule to find $\frac{dy}{dx}$ for the following. (24 pts.)

$y=\left(4x^4-3x^2+4\right)^{-4}$

$\frac{dy}{dx}=-4\left(4x^4-3x^2+4\right)^{-5}\left(16x^3-6x\right)$

+8

$y=\frac{x^3-4x}{(7x-2)^2}$

$\frac{dy}{dx}=\frac{(3x^2-4)(7x-2)^2-(x^3-4x)2(7x-2)^1 7}{(7x-2)^4}$

$=\frac{(3x^2-4)(7x-2)^2-14(x^3-4x)(7x-2)}{(7x-2)^4}$

$=\boxed{\frac{(3x^2-4)(7x-2)-14(x^3-4x)}{(7x-2)^3}}$

+8

$y=9x\left(11x^4+2x\right)^{\frac{2}{3}}$

$\frac{dy}{dx}=9\left(11x^4+2x\right)^{\frac{2}{3}}+9x\left[\frac{2}{3}\left(11x^4+2x\right)^{-\frac{1}{3}}\left(44x^3+2\right)\right]$

$=\boxed{9\left(11x^4+2x\right)^{\frac{2}{3}}+6x\left(44x^3+2\right)\left(11x^4+2x\right)^{-\frac{1}{3}}}$

+8

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7. Find the derivative for the following functions. (16 pts.)

$y = -4e^{-2x+2}$

$\frac{dy}{dx} = -4e^{-2x+2}(-2)$

$= \boxed{8e^{-2x+2}}$ +4

$y = 8^{5x^2-2}$

$\frac{dy}{dx} = \boxed{8^{5x^2-2} \cdot \ln 8 \cdot (10x)}$ +4

$y = \ln(7-8x)$

$\frac{dy}{dx} = \frac{1}{7-8x}(-8) = \boxed{\frac{-8}{7-8x}}$ +4

$y = -\log(x^2+4)$

$\frac{dy}{dx} = -\frac{1}{x^2+4} \cdot \frac{1}{\ln 10} \cdot \frac{2x}{1}$

$= \boxed{\frac{-2x}{(x^2+4)\ln 10}}$ +4

8. The cost function for q units of a certain item is $C(q)=104q+102$. The revenue function for the same item is $R(q)=104q+\frac{55q}{\ln q}$. a) Find the marginal cost. b) Find the profit function. c) Find the profit from one more additional unit sold when 8 units are sold. (8 pts.)

a) $C'(q) = \boxed{104}$ +2

b) $P(q) = R(q) - C(q)$

$= \cancel{104q} + \frac{55q}{\ln q} - \cancel{104q} - 102$

$= \boxed{\frac{55q}{\ln q} - 102}$ +3

c) $P'(q) = \frac{55\ln q - 55q\left(\frac{1}{q}\right)}{(\ln q)^2}$

$= \frac{55\ln q - 55}{(\ln q)^2}$

$P'(8) = \boxed{13.73}$ +3

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