Math 148-AN Hour Exam #1 Satake

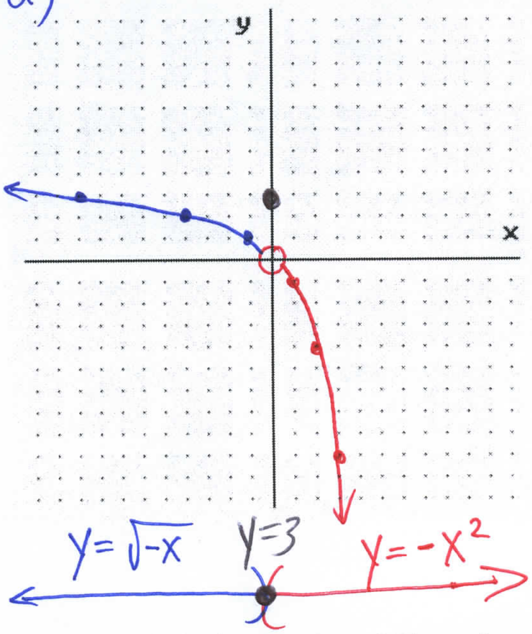
Name: KEY

Read all directions and problems carefully! Show all appropriate work for credit.

1. a) Graph the following piecewise-defined function. b) State the Domain and Range using Interval Notation. (10 pts.)

$$g(x)=\begin{cases}\sqrt{-x} & \text{if } x<0\\ 3 & \text{if } x=0\\ -x^2 & \text{if } x>0\end{cases}.$$

a)

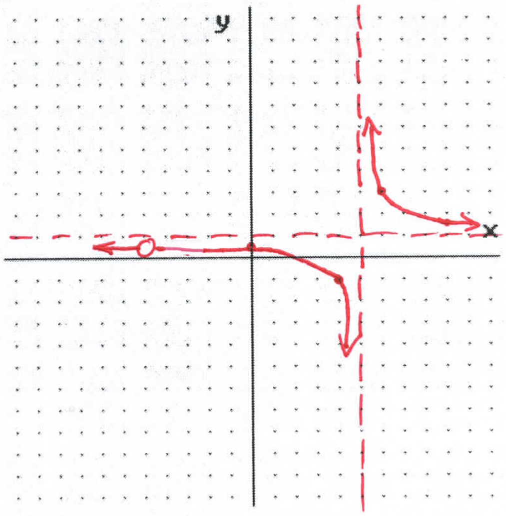


b) $D_g: (-\infty, \infty)$

$R_g: (-\infty, 0) \cup (0, \infty)$

+10

2. a) Graph the Rational Function $r(x)=\dfrac{x^2+2x-15}{x^2-25}$. b) **Label** and sketch any horizontal and vertical asymptotes and any holes that may exist. (10 pts.)



$$r(x)=\frac{\cancel{(x+5)}(x-3)}{(x-5)\cancel{(x+5)}}$$

$$HA: y=\frac{L.C.}{L.C.}=\frac{1}{1} \Rightarrow \boxed{y=1}$$

+10

$$VA: \boxed{x=5}$$

$$\boxed{\text{HOLE @ } x=-5}$$

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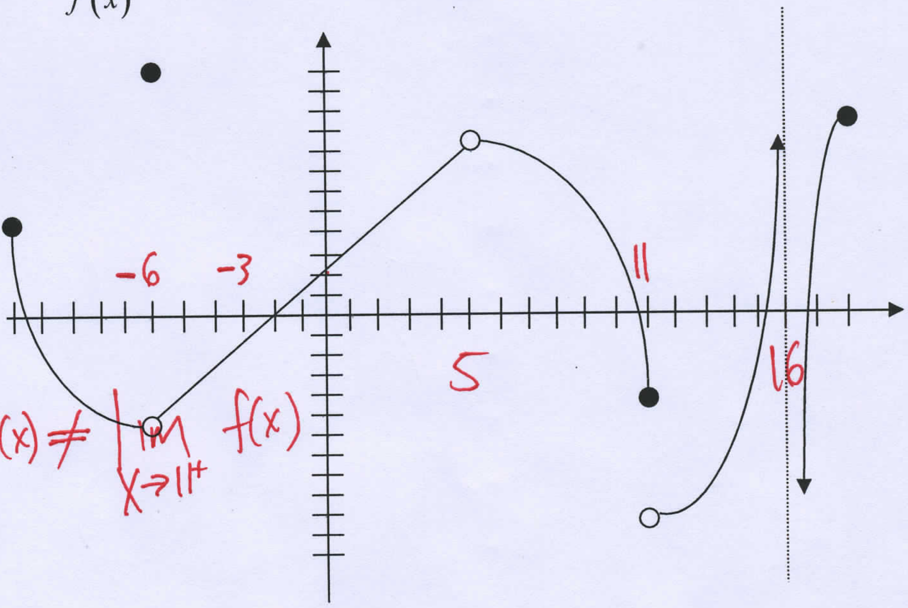
3. Given the graph of the following function, use it to determine the following limits. If the limit does **not** exist, explain why it fails to exist. (15 pts.)

$f(x)$



a) $\lim_{x\to -6} f(x) = \boxed{-6}$ +3

b) $\lim_{x\to -3} f(x) = \boxed{-1}$ +3

c) $\lim_{x\to 5} f(x) = \boxed{8}$ +3

d) $\lim_{x\to 11} f(x) = \boxed{\text{DNE}}$ +3 $\lim_{x\to 11^-} f(x) \neq \lim_{x\to 11^+} f(x)$

e) $\lim_{x\to 16} f(x) = \boxed{\text{DNE}}$ +3 $\lim_{x\to 16^-} f(x) \neq \lim_{x\to 16^+} f(x)$

4. Find the following limits, if the exist, using the functions from Problem #1 and #2. (12 pts.)

a) $\lim_{x\to 0} g(x) = \boxed{0}$ +2

b) $\lim_{x\to -5^+} r(x) = \frac{-5-3}{-5-5} = \frac{-8}{-10} = \boxed{\frac{4}{5}}$ +2

c) $\lim_{x\to -5^-} r(x) = \boxed{\frac{4}{5}}$ +2

d) $\lim_{x\to -5} r(x) = \boxed{\frac{4}{5}}$ +2

e) $\lim_{x\to 5^+} r(x) = \boxed{+\infty}$ +2

f) $\lim_{x\to \infty} r(x) = \boxed{1}$ +2

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5. Use the properties of limits to find the indicated limit. It may be necessary to rewrite the expressions using algebra before the limit properties can be applied. HINT: Conjugate.

(8 pts.) $$\lim_{h\to 0}\frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$$

$$=\lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$$=\frac{1}{\sqrt{x+0}+\sqrt{x}}=\frac{1}{\sqrt{x}+\sqrt{x}}=\boxed{\frac{1}{2\sqrt{x}}}$$ +8

6. Using the graph from Problem #3, determine the points at which the function has discontinuities. For each point identify which conditions for continuity are not met. (15 pts.)

$X=-6$; $\lim_{x\to -6} f(x)\neq f(-6)$ +4

$X=11$; $\lim_{x\to 11} f(x)$ DNE +4

$X=5$; $f(5)$ DNE +4

$X=16$; $f(16)$ DNE +3

7. Find the average rate of change $y=2^x$ for the function between $x=-1$ and $x=4$. (15 pts.)

AVG. RATE OF CHANGE $=\frac{f(b)-f(a)}{b-a}=\frac{f(4)-f(-1)}{4-(-1)}$

$$=\frac{2^4-2^{-1}}{5}=\frac{15.5}{5}=\boxed{3.1}$$ +15

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8. Use the four-step process to find $f'(x)$ and then find $f'(1)$ and $f'(2)$. (15 pts.)

$f(x) = 2x^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2 - 2x^2}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2)-2x^2}{h} = \lim_{h\to 0} \frac{\cancel{2x^2}+4xh+2h^2-\cancel{2x^2}}{h}$$

$$= \lim_{h\to 0} \frac{4xh+2h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(4x+2h)}{\cancel{h}}$$

$$= \lim_{h\to 0} (4x+2h) = 4x+2(0) = \boxed{4x}$$

+15

$$f'(1) = 4(1) = \boxed{4} \; ; \; f'(2) = 4(2) = \boxed{8}$$

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