

Name: KEYRead all directions and problems carefully! Show all appropriate work for credit.

1. Use Integration by Parts to evaluate the following integrals.

$$\int x^2 \ln x \, dx \quad u = \ln x \quad du = \frac{1}{x} dx$$

$$\int x \sqrt{x-1} \, dx \quad u = x \quad du = dx$$

$$dv = (x-1)^{1/2} dx$$

$$v = \frac{2}{3}(x-1)^{3/2}$$

$$= \frac{x^3 \ln x}{3} - \int \frac{x^3}{3x} dx \quad v = \frac{x^3}{3} \quad dv = x^2 dx$$

$$= \frac{2}{3} x (x-1)^{3/2} - \int \frac{2}{3} (x-1)^{3/2} dx$$

$$= \frac{x^3 \ln x}{3} - \int \frac{x^2}{3} dx$$

$$= \frac{2}{3} x (x-1)^{3/2} - \frac{2}{3} \left[\frac{2}{5} (x-1)^{5/2} \right] + C$$

$$= \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$$

$$= \frac{2}{3} x (x-1)^{3/2} - \frac{4}{15} (x-1)^{5/2} + C$$

$$\int x \csc^2 x \, dx \quad u = x \quad du = 1$$

$$\int \tan^{-1} x \, dx \quad \text{let } u = \tan^{-1} x \quad du = \frac{1}{1+x^2} dx$$

$$v = -\cot x \quad dv = \csc^2 x$$

$$\Downarrow \quad v = x \quad dv = dx$$

$$= -x \cot(x) + \int \cot(x) dx$$

$$= x \tan^{-1} x - \int \frac{x}{1+x^2} dx$$

$$= -x \cot(x) + \int \frac{\cos(x)}{\sin(x)} dx \quad u = \sin(x) \quad du = \cos(x)$$

$$\text{let } w = x^2 + 1 \quad dw = 2x dx$$

$$= -x \cot(x) + \int \frac{1}{u} du$$

$$= x \tan^{-1} x - \frac{1}{2} \int \frac{1}{w} dw = x \tan^{-1} x - \frac{1}{2} (\ln|w|) + C$$

$$\boxed{= -x \cot(x) + \ln|\sin(x)| + C}$$

$$\boxed{= x \tan^{-1} x - \frac{1}{2} \ln|x^2 + 1| + C}$$

$$\int x^3 e^x dx \quad [\text{Use Tic Tac Toe}]$$

$$\Rightarrow x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + C$$

x^3	e^x	Sign
$3x^2$	e^x	+
$6x$	e^x	-
6	e^x	+
0	e^x	-