Read all directions and problems carefully! Show all appropriate work for credit.

1. For the function, \( g(x) = -x^3 - 3x^2 + 9x + 15 \):
   
   a) Determine the critical numbers and then write the critical points of the function, \( g(x) \).
   
   b) Write the intervals, using interval notation, on which \( g(x) \) is increasing and decreasing.
   
   c) Use the First Derivative test to locate the local maximum and minimum values. Describe explicitly how this test was used to determine a max or a min.

   d) Determine the inflection numbers and then write the inflection points of the function, \( g(x) \).

   e) Write the intervals, using interval notation, on which \( g(x) \) is concave up and concave down.

\[
\begin{align*}
g'(x) &= -3x^2 - 6x + 9 \\
g''(x) &= -6x - 6 \\
g(3) &= -3(3)^2 + 3(9) + 15 = 72 \\
g'(3) &= -3(3)^2 + 3(9) + 15 = 72 \\
g''(3) &= -6(3) - 6 = -24 \\
x &= -1 \\
\end{align*}
\]
Math 148-AN

Quiz #5

Name:

Read all directions and problems carefully! Show all appropriate work for credit.

1. For the function, \( g(x) = -x^3 - 3x^2 + 9x + 15; \)
   
   a) Determine the critical numbers and then write the critical points of the function, \( g(x). \)
   
   b) Write the intervals, using interval notation, on which \( g(x) \) is increasing and decreasing.
   
   c) Use the First Derivative test to locate the local maximum and minimum values. Describe explicitly how this test was used to determine a max or a min.
   
   d) Determine the inflection numbers and then write the inflection points of the function, \( g(x). \)
   
   e) Write the intervals, using interval notation, on which \( g(x) \) is concave up and concave down.

\[
\begin{align*}
g(x) &= -x^3 - 3x^2 + 9x + 15 \\
g'(x) &= -3x^2 - 6x + 9 \\
g''(x) &= -6x + 6
\end{align*}
\]

\[
\begin{align*}
g''(x) &= 0 \\
x &= \frac{-(-6)}{2(-6)} = \frac{6}{12} = 1
\end{align*}
\]

\[
\begin{align*}
\text{Critical points:} & \quad (-3, 1) \\
\text{Sign chart:} & \quad (-\infty, -3) \quad \bigcup \quad (-3, 1) \quad \bigcup \quad (1, \infty)
\end{align*}
\]

\[
\begin{align*}
\text{Increasing on:} & \quad (-\infty, -3) \quad \bigcup \quad (1, \infty) \\
\text{Decreasing on:} & \quad (-3, 1)
\end{align*}
\]

\[
\begin{align*}
\text{Concave up on:} & \quad (-\infty, -1) \quad \bigcup \quad (1, \infty) \\
\text{Concave down on:} & \quad (-1, 1)
\end{align*}
\]
Read all directions and problems carefully! Show all appropriate work for credit.

1. For the function, \( g(x) = -x^3 - 3x^2 + 9x + 15 \):
   a) Determine the critical numbers and then write the critical points of the function, \( g(x) \).
   b) Write the intervals, using interval notation, on which \( g(x) \) is increasing and decreasing.
   c) Use the First Derivative test to locate the local maximum and minimum values. Describe explicitly how this test was used to determine a max or a min.
   d) Determine the inflection numbers and then write the inflection points of the function, \( g(x) \).
   e) Write the intervals, using interval notation, on which \( g(x) \) is concave up and concave down.

\[
g(x) = -x^3 - 3x^2 + 9x + 15
\]
\[
g'(x) = -3x^2 - 6x + 9
\]
\[
= -3(x^2 + 2x - 3)
\]
\[
= -3(x-1)(x+3)
\]
\[
x = 1 \quad \text{and} \quad x = -3
\]

A) Critical numbers: \( x = -3 \) and \( x = 1 \)

Critical points: \((1, 20)\), \((-3, -12)\)

\[
g(1) = -(1)^3 - 3(1)^2 + 9(1) + 15
\]
\[
= -1 - 3 + 9 + 15
\]
\[
g(1) = 20
\]

\[
g(-3) = -(3)^3 - 3(-3)^2 + 9(-3) + 15
\]
\[
= 27 - 27 - 27 + 15
\]
\[
g(-3) = -12
\]

B) \((-\infty, -3) U (1, \infty)\): Decreasing

\((-3, 1)\): Increasing

C) Increasing:

\((-\infty, -3)\)

\((1, \infty)\)

Decreasing:

\((-3, 1)\)

D) Relative maximum at \((1, 20)\) when \( x = 1 \).

Relative minimum at \((-3, -12)\) when \( x = -3 \).

This test would be used to determine a max or a min by when \( g''(x) \) goes from negative to positive, you have a min. When \( g''(x) \) goes from positive to negative, you have a max.

\[
g''(x) = -6x - 6
\]
\[
= 6(x+1) = 0
\]
\[
x = -1
\]

Inflection Numbers: \( x = -1 \)

\[
-((-1)^3 - 3(-1)^3 + 9(-1) + 15
\]
\[
1 - 3 - 9 + 15 = 4
\]